OCR
Oxford Cambridge and RSA

Wednesday 22 May 2019 – Morning

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

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INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \text{ or } \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

1 **In this question you must show detailed reasoning.**

Solve the equation $x(3-\sqrt{5})=24$, giving your answer in the form $a+b\sqrt{5}$, where a and b are positive integers. **[3]**

$\Rightarrow x = \frac{24}{3-\sqrt{5}}$ (Rationlise denominator)

$x = \frac{24}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$

$= \frac{72+24\sqrt{5}}{9-5+3\sqrt{5}-3\sqrt{5}}$

$= \frac{72+24\sqrt{5}}{4}$

$= 18+6\sqrt{5}$

2 **(a)** Express $5x^2-20x+3$ in the form $p(x+q)^2+r$, where p, q and r are integers. **[3]**

(b) State the coordinates of the minimum point of the curve $y=5x^2-20x+3$. **[2]**

(c) State the equation of the normal to the curve $y=5x^2-20x+3$ at its minimum point. **[1]**

a. $5x^2-20x+3$

$=5(x^2-4)+3$

$=5((x-2)^2-2^2)+3$

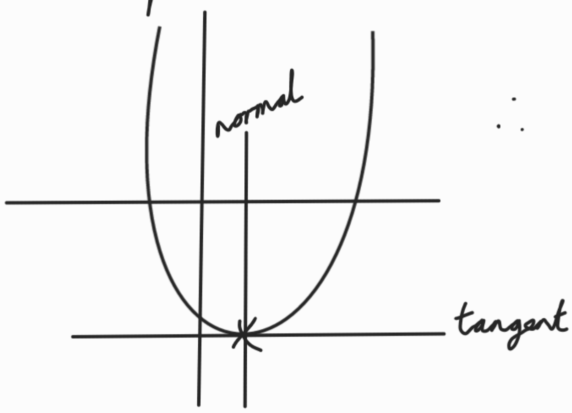
$=5(x-2)^2-20+3$

$=5(x-2)^2-17$

b. if $y = p(x+q)^2 + r$, minimum point $= (-q, r)$

$\therefore$ minimum point for $y = 5(x-2)^2 - 17$ is $\underline{(2, -17)}$

c. Shape of curve:



$\therefore$ normal $x = 2$

3 **(a)** Sketch the curve $y = -\dfrac{1}{x^2}$. **[1]**

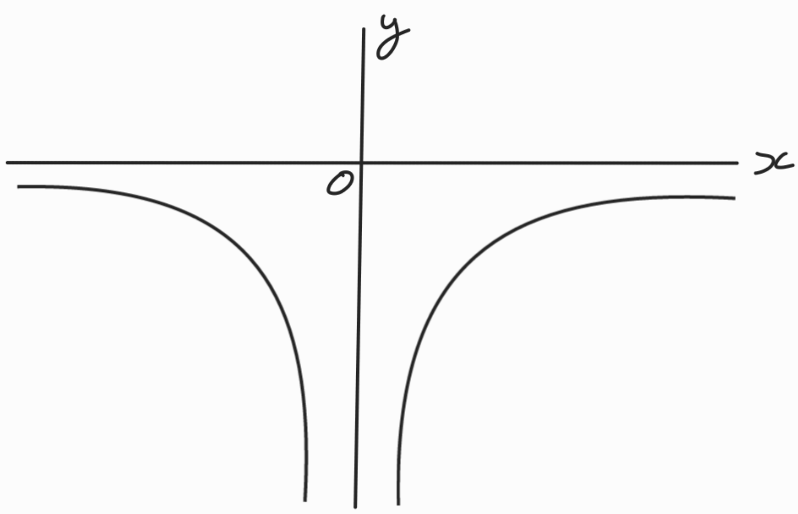
(b) The curve $y = -\dfrac{1}{x^2}$ is translated by 2 units in the positive x-direction.

State the equation of the curve after it has been translated. **[2]**

(c) The curve $y = -\dfrac{1}{x^2}$ is stretched parallel to the y-axis with scale factor $\frac{1}{2}$ and, as a result, the point $\left(\frac{1}{2}, -4\right)$ on the curve is transformed to the point P.

State the coordinates of P. **[2]**

a.



· Axes are asymptotes.
· Symmetrical

b. $y=f(x)$ translated a units in positive x - axis : $y=f$

$\therefore\ y = -\dfrac{1}{x^2} \rightarrow y = \dfrac{-1}{(x-2)^2}$

c. Stretch in y-axis $\rightarrow$ y-coord $\times \frac{1}{2}$, x-coord $\times 1$

$\therefore\ \left(\frac{1}{2}, -4\right) \rightarrow \left(\frac{1}{2}, -2\right)$

4 **(a)** Find and simplify the first three terms in the expansion of $(2-5x)^5$ in ascending powers of x. **[3]**

(b) In the expansion of $(1+ax)^2(2-5x)^5$, the coefficient of x is 48.

Find the value of a. **[3]**

a. Pascal's triangle :

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$\therefore (2-5x)^5 \approx 2^5 + 5\times 2^4 \times(-5x) + 10\times 2^3 \times (-5x)^2$

$= 32 - 400x + 2000x^2$

b. $(1+ax)^2 = 1 + 2ax + a^2x^2$

$\therefore (1+ax)^2(2-5x)^5 \approx (1+2ax+a^2x^2)(32-400x+2000x^3)$

(ignore x^2 terms)

$\Rightarrow 64a - 400 = 48$

$\Rightarrow \underline{a = 7}$

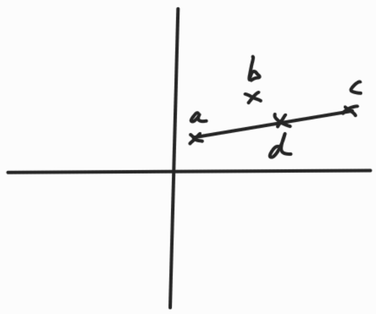
5 Points A, B, C and D have position vectors $\mathbf{a} = \begin{pmatrix}1\\2\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}3\\5\end{pmatrix}$, $\mathbf{c} = \begin{pmatrix}7\\4\end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix}4\\k\end{pmatrix}$.

(a) Find the value of k for which D is the midpoint of AC. **[1]**

(b) Find the two values of k for which $|\overrightarrow{AD}| = \sqrt{13}$. **[3]**

(c) Find one value of k for which the four points form a trapezium. **[2]**

a.



$$\bar{y}_d = k = \frac{\bar{y}_c + \bar{y}_a}{2} = \frac{4+2}{2} = \underline{3}$$

b. $\sqrt{(1-4)^2 + (2-k)^2} = \sqrt{13}$

$(1-4)^2 + (2-k)^2 = 13$

$9 + 4 + k^2 - 4k = 13$

$k^2 - 4k = 0$

$k(k-4) = 0$

$\Rightarrow k = 0$ or $k = 4$

c. Trapezium $\Rightarrow$ two sides are parallel
$\Rightarrow$ equal gradients

$\therefore$ gradient of $\overrightarrow{CA}$ = gradient of $\overrightarrow{DB}$

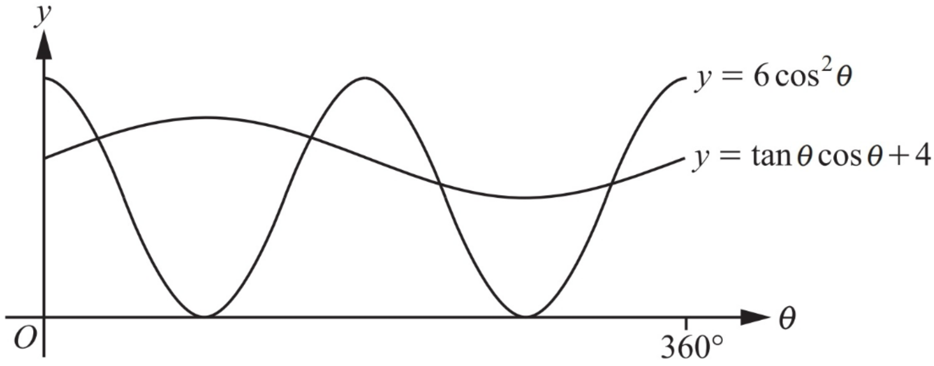
$$\frac{4-2}{7-1} = \frac{k-5}{4-3} \Rightarrow k = \underline{\frac{16}{3}}$$

6 In this question you must show detailed reasoning.

(a) Show that the equation $6\cos^2\theta = \tan\theta\cos\theta + 4$

can be expressed in the form $6\sin^2\theta + \sin\theta - 2 = 0$. **[2]**

(b)



The diagram shows parts of the curves $y = 6\cos^2\theta$ and $y = \tan\theta\cos\theta + 4$, where θ is in degrees.

Solve the inequality $6\cos^2\theta > \tan\theta\cos\theta + 4$ for $0° < \theta < 360°$. **[5]**

a. $\cos^2\theta = 1 - \sin^2\theta$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

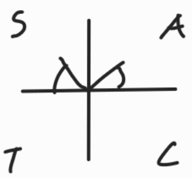
$\therefore\ 6(1 - \sin^2\theta) = \dfrac{\sin\theta}{\cos\theta}(\cos\theta) + 4$

$6 - 6\sin^2\theta = \sin\theta + 4$

$6\sin^2\theta + \sin\theta - 2 = 0$

b. Solving quadratic from (a): $\sin\theta = \frac{1}{2}$ or $-\frac{2}{3}$

$\sin\theta = \frac{1}{2}$:

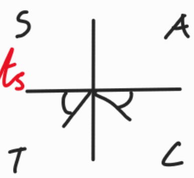


$\sin^{-1}\left(\frac{1}{2}\right) = 30$

$\theta = 30°$ ← these are points

$\theta = 150°$ of intersection

$\sin\theta = -\frac{2}{3}$



$\sin^{-1}\left(\frac{2}{3}\right) = 41.8$

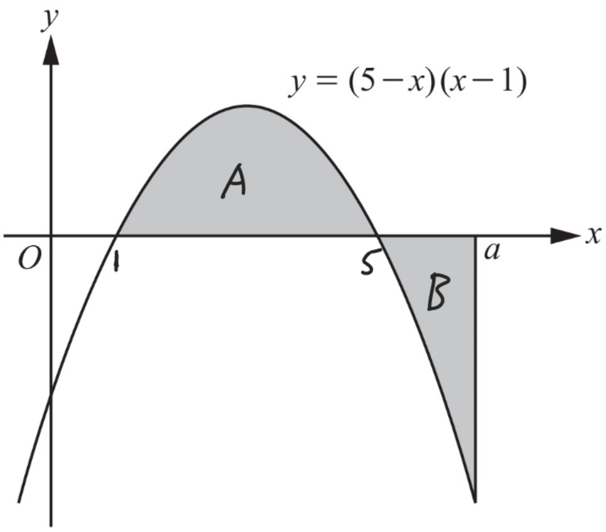
$\theta = 221.8°$

$\theta = 318.2°$

$\therefore$ intervals : $0 < \theta < 30$
$150 < \theta < 222$
$318 < \theta < 360$

(These are the interva[ls of] the graph where $y = 6\cos^2 x$ is above $y = \tan\theta\cos\theta + 4$)

7



The diagram shows part of the curve $y = (5-x)(x-1)$ and the line $x = a$.

Given that the total area of the regions shaded in the diagram is 19 units^2, determine the exact value of a. **[8]**

$y = (5-x)(x-1) = 5x - 5 - x^2 + x = -x^2 + 6x - 5$

at $y = 0$, $x = 5$ or $x = 1$

$\therefore$ area B $= \int_5^a (-x^2 + 6x - 5)\,dx$

$= \left[-\frac{x^3}{3} + 3x^2 - 5x\right]_5^a$

$= \left(-\frac{a^3}{3} + 3a^2 - 5a\right) - \left(-\frac{5^3}{3} + 3\times 5^2 - 5\times 5\right)$

$= -\frac{a^3}{3} + 3a^2 - 5a + \frac{25}{3}$

area $A = \int_1^5 (-x^2 + 6x - 5)\,dx$

$$= \left[-\frac{x^3}{3} + 3x^2 - 5x\right]_1^5$$

$$= -\frac{5^3}{3} + 3\times5^2 - 5\times5 - \left(-\frac{1^3}{3} + 3\times1^2 - 5\right)$$

$$= \frac{32}{3}$$

$$\Rightarrow 19 = \frac{32}{3} - \frac{a^3}{3} + 3a^2 - 5a + \frac{25}{3}$$

(×3 and rearrange)

$$a^3 - 9a^2 + 15a = 0$$

$$\Rightarrow a^2 - 9a + 15 = 0$$

$$\therefore a = \frac{9 \pm \sqrt{21}}{2}$$

however $a > 5$, so $a = \dfrac{9 + \sqrt{21}}{2}$

8 **(a)** Show that the equation $2\log_2 x = \log_2(kx-1)+3$, where k is a constant, can be expressed in the form $x^2-8kx+8=0$. **[4]**

(b) Given that the equation $2\log_2 x = \log_2(kx-1)+3$ has only one real root, find the value of this root. **[4]**

a. $2\log_2 x = \log_2(kx-1)+3$

$\log_2 x^2 - \log_2(kx-1) = 3$

Using laws of logs

$\log_2\left(\frac{x^2}{kx-1}\right) = 3$

$\frac{x^2}{kx-1} = 2^3$

(rewrite both sides as the powers of 2 to remove logs)

$x^2 = 8(kx-1)$

$x^2 - 8kx + 8 = 0$

b. One real root $\Rightarrow b^2-4ac=0$

$(-8k)^2 - 4\times1\times8 = 0$

$64k^2 = 32$

$k^2 = 1/2$

$k = \pm 1/\sqrt{2}$

$\therefore x = \pm 2\sqrt{2}$

However $\log_2 x$ is only defined for $x > 0$

$\therefore x = 2\sqrt{2}$

9 Three forces $\begin{pmatrix} 7 \\ -6 \end{pmatrix}$N, $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$N and **F** N act on a particle.

Given that the particle is in equilibrium under the action of these three forces, calculate **F**. **[2]**

Equilibrium $\Rightarrow$ total in each direction $= 0$

Let $\underline{F} = \begin{pmatrix} a \\ b \end{pmatrix}$

Resolve in x direction:

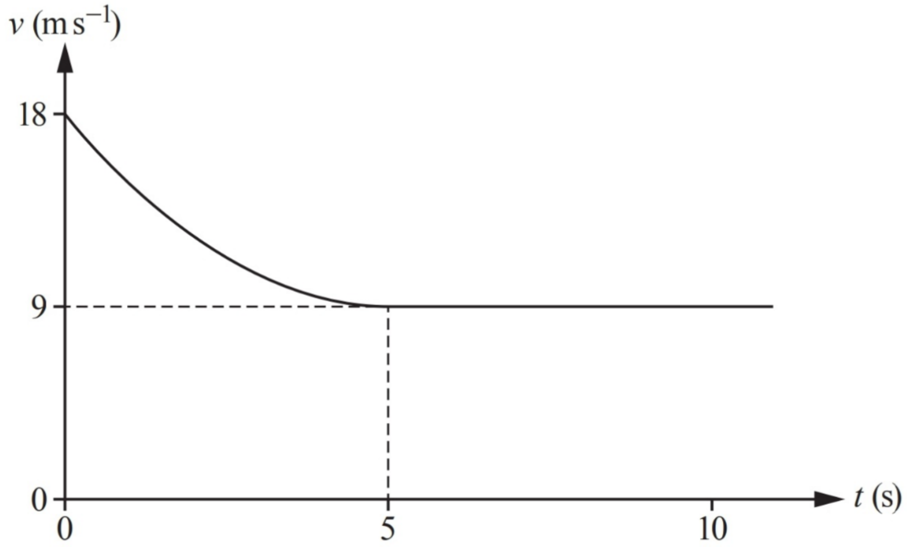
$7 + 2 + a = 0 \quad \Rightarrow a = -9$

Resolve in y direction:

$-6 + 5 + b = 0 \quad \Rightarrow b = 1$

$\therefore \underline{F} = \begin{pmatrix} -9 \\ 1 \end{pmatrix} \text{N}$

10



The diagram shows the velocity-time graph modelling the velocity of a car as it approaches, and drives through, a residential area.

The velocity of the car, v m s^{-1}, at time t seconds for the time interval $0 \leqslant t \leqslant 5$ is modelled by the equation $v = pt^2 + qt + r$, where p, q and r are constants.

It is given that the acceleration of the car is zero at $t = 5$ and the speed of the car then remains constant.

(a) Determine the values of p, q and r. **[5]**

(b) Calculate the distance travelled by the car from $t = 2$ to $t = 10$. **[3]**

a. $v = pt^2 + qt + r$

at $t = 0$, $v = 18 \text{ ms}^{-1}$

$\Rightarrow \underline{r = 18}$

at $t = 5$, $v = 9 \text{ ms}^{-1}$

$\Rightarrow 9 = 25p + 5q + 18$

$25p + 5q + 9 = 0$ (1)

at $t = 5$, $\frac{dv}{dt} = a = 0$

$\frac{dv}{dt} = 2pt + q \quad \Rightarrow \quad 0 = 10p + q \quad \Rightarrow \quad q = -10p$ (2)

Sub ② into ①:

$$25p - 50p + 9 = 0$$
$$p = \frac{9}{25}$$

$$\Rightarrow q = -\frac{90}{25}$$
$$q = -\frac{18}{5}$$

b. $x = \int_a^b v \, dt$

$$= \int_2^5 \left(\frac{9}{25} t^2 - \frac{18}{5} t + 18 \right) dt$$

$$= \left[\frac{3}{25} t^3 - \frac{9}{5} t^2 + 18t \right]_2^5$$

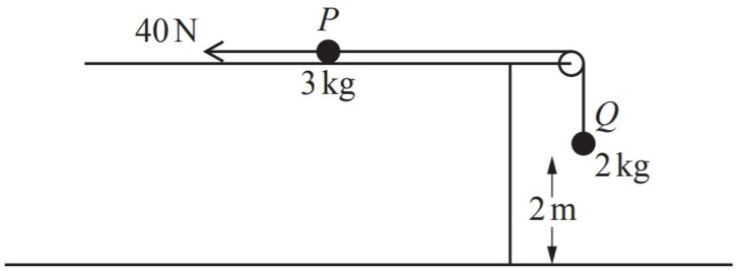
$$= \left(\frac{3}{25} \times 125 - \frac{9}{5} \times 25 + 18 \times 5 \right) - \left(\frac{3}{25} \times 8 - \frac{9}{5} \times 4 + 18 \times 2 \right)$$

$$= 60 - \frac{744}{25} = 30.24 \text{ m}$$

x from 10 to 5 $= s \times t = 9 \times 5 = 45$ m

$$\Rightarrow \Sigma x = 30.24 + 45$$
$$= 75.24 \text{ m}$$

11 Two small balls P and Q have masses 3 kg and 2 kg respectively. The balls are attached to th of a string. P is held at rest on a rough horizontal surface. The string passes over a pulley which is fixed at the edge of the surface. Q hangs vertically below the pulley at a height of 2 m above a horizontal floor.



The system is initially at rest with the string taut. A horizontal force of magnitude 40 N acts on P as shown in the diagram.

P is released and moves directly away from the pulley. A constant frictional force of magnitude 8 N opposes the motion of P. It is given that P does not leave the horizontal surface and that Q does not reach the pulley in the subsequent motion.

The balls are modelled as particles, the pulley is modelled as being small and smooth, and the string is modelled as being light and inextensible.

(a) Show that the magnitude of the acceleration of each particle is $2.48\,\mathrm{m\,s^{-2}}$. **[5]**

(b) Find the tension in the string. **[2]**

When the balls have been in motion for 0.5 seconds, the string breaks.

(c) Find the additional time that elapses until Q hits the floor. **[5]**

(d) Find the speed of Q as it hits the floor. **[2]**

(e) Write down the magnitude of the normal reaction force acting on Q when Q has come to rest on the floor. **[1]**

(f) State one improvement that could be made to the model. **[1]**

a. Newton II for P:

$40 - T - 8 = 3a$ ①

Newton II for Q:

$T - 2g = 2a$

$\Rightarrow T = 2a + 2g$ ②

Sub ② into ①:

$$40 - 2a - 2g - 8 = 3a$$

$$a = 2.48 \text{ ms}^{-2}$$

b. $T = 2 \times 2.48 + 2g$

$= 24.6 \text{ N}$

c. speed $= 2.48 \times 0.5 = 1.24 \text{ ms}^{-1}$

distance travelled in this 0.5 s:

$x = x$
$u = 0$
$v = 1.24$
$a = 2.48$
$t = 0.5$

$$x = ut + \frac{1}{2}at^2$$
$$= 0 + \frac{1}{2} \times 2.48 \times 0.5^2$$
$$= 0.31 \text{ m}$$

After string breaks:

$x = -(2 + 0.31) = -2.31 \text{ m}$
$u = 1.24$
$v =$
$a = -9.8$
$t = t$

$$x = ut + \frac{1}{2}at^2$$
$$-2.31 = 1.24t + \frac{1}{2} \times -9.8t^2$$

$$4.9t^2 + 1.24t - 2.31 = 0$$
$$\Rightarrow t = \underline{0.825 \text{ s}}$$

d. $x = -2.31$ $\quad v^2 = u^2 + 2ax$

$u = 1.24$

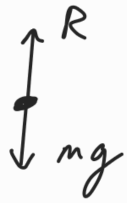
$v = v$ $\quad v^2 = 1.24^2 + 2(-9.8)(-2.31)$

$a = -9.8$

$t = 0.825$ $\quad v = \sqrt{46.79}$

$v = 6.84 \text{ ms}^{-1}$

e. Rest $\Rightarrow \Sigma f = 0$



$\therefore R = mg = 19.6 \text{ N}$

f. One of :

- More accurate g value
- Model resistance as variable
- Include dimensions of pulley
- Include the friction in the pulley